Solution of 2D Electromagnetic Scattering Using IEFG-UPML Method

Leonardo B. Lopes[1], Ursula C. Resende[2], Sandro T. Gonçalves[3]
Departament of Electrical Engineering, Centro Federal de Educação Tecnológica de Minas Gerais, 30510-000 Brazil
[1]leobruno@gmail.com [2]resendeursula@des.cefetmg.br [3]sandro@div.cefetmg.br

This paper presents a new approach for solution of 2D electromagnetic scattering using interpolating element-free galerkin method with perfectly matched layers technique in order to limit the computational domain. The uniaxial perfectly matched layers in cylindrical coordinates and the first-order Bayliss-Turkel absorbing aboundary conditions are used to limit the computational domain. It is considered the *TMz* plane wave scattering by a *z*-infinite dielectric cylinder and the results obtained from proposed technique are compared with analytical solution.

***Index Terms*—electromagnetic scattering, element-free Galerkin method, meshless methods, uniaxial perfect matched layers.**

I. Introduction

MESHLESS methods (MM) are a new class of numerical methods that has been used for solution of partial differential equations (PDE). It does not require a mesh structure and the solution is obtained using only a cloud of nodes spread throughout the region of interest. This feature makes MM appropriate to deal with complex geometries and inhomogeneities. These methods were firstly used to solve problems related to structures and fluid mechanics. It recently has been successfully applied in electromagnetic problems [1], [2]. However, for solution of electromagnetic scattering, differential equation based methods can not incorporate the Sommerfeld radiation condition. Then it is necessary to extend the discretization domain by establishing a fictitious boundary at some distance away from the scatter, where this condition is approximately imposed [1], [3], [4]. This process leads to a significant increase of computational requirements. In order to bring the fictitious boundary close to the scatter, fictitious absorbers especially designed to have high attenuation and zero reflection coefficient have been used in an approximation called perfectly matched layers (PML) [5]. An improvement of this approach, known as uniaxial PML (UPML) employs anisotropic material properties to describe the absorbing layers. These techniques have been successfully used with finite element method (FEM) and finite difference time domain (FDTD) for solving electromagnetic scattering [6].

Among the MM available in the literature, the interpolating element-free Galerkin method (IEFG) is one of the most investigated and used methods because of its robustness and good convergence rates [7]. In this work IEFG and UPML are combined and presented as a new method for solving 2D electromagnetic scattering problems.

II. Electromagnetic Scattering Problem Modeling

The problem investigated in this work is the TMz plane wave scattering by a z-directed infinitely long dielectric cylinder Ω_1 with relative electric permittivity ϵ_r, relative magnetic permeability μ_r and surrounded by free space Ω_2, as illustrated in Fig. 1. So the 2D domain is $\Omega = \Omega_1 \cup \Omega_2$ and Γ is the global

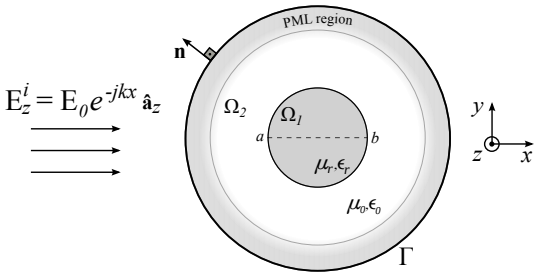

Fig. 1. 2D scattering problem.

boundary in which the normal vector is $\mathbf{n}$. The total electric field E_z, which has only the z-component, is calculated by the bi-dimensional scalar Helmholtz equation $\nabla^2 E_z + k_0^2 E_z = 0$, where k_0 is the vacuum wave number [1].

The weak form of the problem, obtained using the method of weighted residuals with test function w and the first-order Bayliss-Turkel ABC applied to Γ, is [8].

$$\int_\Omega \left[\nabla w\left(\mu_r^{-1}\nabla E_z\right) - k_0^2 \epsilon_r w E_z\right] d\Omega + \int_\Gamma \gamma w E_z d\Gamma = \int_\Gamma q w d\Gamma, \quad (1)$$

where $\gamma = \mu^{-1}\left[jk_0 + (2\rho_r)^{-1}\right]$, $q = \mu^{-1}\left(\nabla E_z^i \cdot \mathbf{n} + \gamma E_z^i\right)$ and E_z^i is the incident electric field.

III. IEFG Formulation

In the IEFG approach, each node I is a point $\mathbf{x}_I = (x, y) \in \Omega$ for which a shape function $\Phi_I(\mathbf{x}_I)$ is associated. $\Phi_I = 0$ for the whole domain Ω, except a region near $\mathbf{x}_I$. Thus, the unknown function can be approximated by [9]:

$$E_z(\mathbf{x}) = \sum_{I=1}^{N} \Phi_I(\mathbf{x}_I) v_I, \quad (2)$$

where N is the number of nodes in Ω, $\mathbf{x}_I$ and v_I are unknown coefficients of node I.

The IEFG uses the interpolant minimum least squares (IMLS) approximation which makes EFG Kronecker delta compatible. The coefficients v_I are determined by minimizing a weighted discrete L^2 norm, with singular weight function $W(r_I) = (r_I^n + \beta^n)^{-1}$ [1], where β is a constant small enough

to ensure no division by zero, n is a constant adjusted to improve the result accuracy, $r_I = |\mathbf{x} - \mathbf{x}_I|/d_I$, $d_I = \alpha d_C$ is the support of the weight function, α is a scaling factor for the influence domain and d_C is the nodal radius [9].

IV. Uniaxial PML

The UPML approach uses anisotropic material properties to describe the absorbing layers. Its main advantage is that it does not require any modification of Maxwell's equations [6].

In rectangular coordinate system, the material parameters $\bar{\bar{\epsilon}}$ and $\bar{\bar{\mu}}$ of the UPML region are an uniaxial tensor [6]. In cylindrical coordinates, this tensor has the following form [10]:

$$\frac{\bar{\bar{\epsilon}}}{\epsilon} = \frac{\bar{\bar{\mu}}}{\mu} = \begin{bmatrix} \frac{\tilde{\rho}}{\rho}\frac{1}{s(\rho)} & & \\ & \frac{\rho}{\tilde{\rho}}s(\rho) & \\ & & \frac{\tilde{\rho}}{\rho}s(\rho) \end{bmatrix}, \tag{3}$$

where $s(\rho) = \delta(\rho)[1 + j\eta_0\sigma(\rho)]$ and $\tilde{\rho} = \rho_0 + \int_{\rho_0}^{\rho} s(\rho')d\rho'$, with $\delta(\rho) = 1 + \delta_{max}(\rho/L)^m$, $\sigma(\rho) = \sigma_{max}(\rho/L)^m$. L is the number of layers in PML region, ρ_0 is the first layer, ρ is the layer distance to center, σ_{max}, δ_{max} and m are adjustable parameters.

In practical implementations of the UPML the absorbing region must be truncated [6] using a PEC backing in the last layer or by applying an ABC condition on the PML region boundary. Here, the IEFG is used along with a first order Bayliss-Turkel ABC applied to Γ.

V. Numerical Results

The proposed technique is applied for solving the electromagnetic scattering by a 0.3λ dielectric cylinder with relative permittivity $\epsilon_r = 2$ and the vacuum wavelength $\lambda = 1m$. The numeric solution (NS) calculated by IEFG algorithm is checked against the analytical solution (AS) by the following L^2 norm error over the whole domain Ω [1]:

$$EL^2 = \sqrt{\iint_\Omega \left|E_z^{\text{NS}} - E_z^{\text{AS}}\right|^2 d\Omega} \Big/ \sqrt{\iint_\Omega |E_z^{\text{AS}}|^2 d\Omega} \times 100\%. \tag{4}$$

The PML parameters are set to $\sigma_{max} = 1$, $a_{max} = 3$ and $m = 5$. The algorithm is tested in two cases: number of layers $L = 10$ and $L = 20$. The results – sampled in the diametrical a-b line across Ω_1 – are shown in Fig. 2 and Fig. 3.

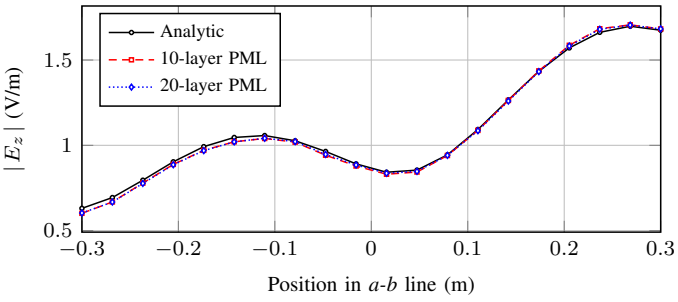

Fig. 2. Electric field magnitude

The achieved results show that, in both cases, the new proposed approach presents a very good accuracy when compared to the analytical solution. The L^2 norm error in the first case, with a 10-layer PML, is $EL^2 = 1.62\%$, and for the 20-layer case is $EL^2 = 1.59\%$. The increase of layer number leads to practically the same result. However, in this case the computational cost is 30% higher.

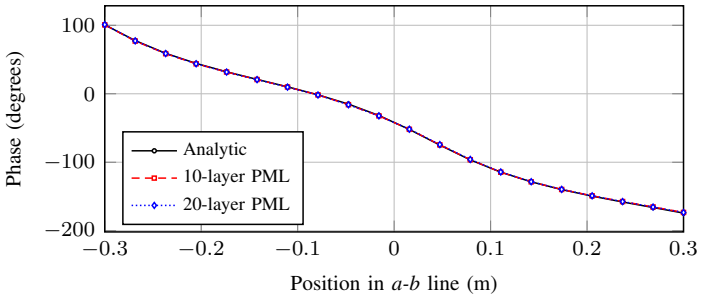

Fig. 3. Electric field phase

VI. Conclusion

This paper presents a new numeric technique for the solution of electromagnetic scattering. This technique is based on the IEFG meshless method which implements the interpolant minimum least squares to provide a Kronecker delta compatible method. The IEFG is coupled to the UPML method which is applied in conjunction with a first order ABC to provide a high performance domain limitation. The numeric results are compared to the analytic ones by the L^2 norm error.

Comparisons with other numerical methods and results for other kinds of 2D scatters will be presented in the full version of the paper.

Acknowledgment

This work was partially supported by FAPEMIG, CAPES, CNPq, and CEFET-MG.

References

[1] U. C. Resende, E. H. R. Coppoli, and M. M. Afonso, "A meshless approach using efg interpolating moving least-squares method in 2-d electromagnetic scattering analysis," *IEEE Trans. Magn.*, vol. 51, no. 3, pp. 1–4, Mar. 2015.

[2] A. Manzin and O. Bottauscio, "Element-free galerkin method for the analysis of electromagnetic-wave scattering," *IEEE Trans. Magn.*, vol. 44, no. 6, pp. 1366–1369, Jun. 2008.

[3] B. Engquist and A. Majda, "Absorbing boundary conditions for the numerical simulation of waves," *Mathematics of Computation*, vol. 31, no. 139, pp. 629–651, Jul. 1977.

[4] A. Bayliss and E. Turkel, "Radiation boundary conditions for wave-like equations," *Commun. Pure Appl. Math.*, vol. 33, no. 6, pp. 707–725, Nov. 1980.

[5] J.-P. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *Journal of Computational Physics*, vol. 114, no. 2, pp. 185–200, 1994.

[6] Z. S. Sacks, D. M. Kingsland, R. Lee, and J.-F. Lee, "A perfectly matched anisotropic absorber for use as an absorbing boundary condition," *IEEE Trans. Antennas Propag.*, vol. 43, no. 12, pp. 1460–1463, Dec. 1995.

[7] T. Belytschko, Y. Y. Lu, and L. Gu, "Element-free galerkin methods," *Int. J. Numer. Meth. Eng.*, vol. 37, no. 2, pp. 229–256, Jan. 1994.

[8] A. F. Peterson, S. L. Ray, and R. Mittra, *Computational methods for electromagnetics*, 1st ed., I. Press, Ed. New York, NY: Oxford University Press, 1998.

[9] G. R. Liu, *Meshfree methods: moving beyond the finite element method*, 2nd ed. Boca Raton: CRC Press, 2010.

[10] J. Maloney, M. Kesler, and G. Smith, "Generalization of pml to cylindrical geometries," *13th Annual Review of Progress in Applied Computational Electromagnetics*, vol. 2, pp. 900–908, 1997.